

INVERSE TRIGONOMETRIC FUNCTION PDF

1. Principal Values & Domains of Inverse Trigonometric/Circular Functions:

	Function	Domain	Range
(i)	$y = \sin^{-1} x$ where	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
(ii)	$y = \cos^{-1} x$ where	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
(iii)	$y = \tan^{-1} x$ where	$x \in \mathbb{R}$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
(iv)	$y = \operatorname{cosec}^{-1} x$ where	$x \leq -1$ or $x \geq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$
(v)	$y = \sec^{-1} x$ where	$x \leq -1$ or $x \geq 1$	$0 \leq y \leq \pi; y \neq \frac{\pi}{2}$
(vi)	$y = \cot^{-1} x$ where	$x \in \mathbb{R}$	$0 < y < \pi$

- P - 2
- (i) $\sin^{-1}(\sin x) = x, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
 - (ii) $\cos^{-1}(\cos x) = x; \quad 0 \leq x \leq \pi$
 - (iii) $\tan^{-1}(\tan x) = x; \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$
 - (iv) $\cot^{-1}(\cot x) = x; \quad 0 < x < \pi$
 - (v) $\sec^{-1}(\sec x) = x; \quad 0 \leq x \leq \pi, x \neq \frac{\pi}{2}$
 - (vi) $\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x; \quad x \neq 0, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
- P - 3
- (i) $\sin^{-1}(-x) = -\sin^{-1} x, \quad -1 \leq x \leq 1$
 - (ii) $\tan^{-1}(-x) = -\tan^{-1} x, \quad x \in \mathbb{R}$
 - (iii) $\cos^{-1}(-x) = \pi - \cos^{-1} x, \quad -1 \leq x \leq 1$
 - (iv) $\cot^{-1}(-x) = \pi - \cot^{-1} x, \quad x \in \mathbb{R}$
- P - 5
- (i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \quad -1 \leq x \leq 1$
 - (ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \quad x \in \mathbb{R}$
 - (iii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}, \quad |x| \geq 1$

2. Identities of Addition and Subtraction:

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$$\begin{aligned} \text{(i)} \quad & \sin^{-1} x + \sin^{-1} y \\ &= \sin^{-1} \left[x \sqrt{1-y^2} + y \sqrt{1-x^2} \right], \quad x \geq 0, y \geq 0 \text{ \& } (x^2 + y^2) \leq 1 \\ &= \pi - \sin^{-1} \left[x \sqrt{1-y^2} + y \sqrt{1-x^2} \right], \quad x \geq 0, y \geq 0 \text{ \& } x^2 + y^2 > 1 \\ \text{(ii)} \quad & \cos^{-1} x + \cos^{-1} y = \cos^{-1} \left[xy - \sqrt{1-x^2} \sqrt{1-y^2} \right], \quad x \geq 0, y \geq 0 \\ \text{(iii)} \quad & \tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}, \quad x > 0, y > 0 \text{ \& } xy < 1 \\ &= \pi + \tan^{-1} \frac{x+y}{1-xy}, \quad x > 0, y > 0 \text{ \& } xy > 1 \\ &= \frac{\pi}{2}, \quad x > 0, y > 0 \text{ \& } xy = 1 \end{aligned}$$

$$\text{I - 2 (i)} \quad \sin^{-1} x - \sin^{-1} y = \sin^{-1} \left[x\sqrt{1-y^2} - y\sqrt{1-x^2} \right], \quad x \geq 0, y \geq 0$$

$$\text{(ii)} \quad \cos^{-1} x - \cos^{-1} y = \cos^{-1} \left[xy + \sqrt{1-x^2} \sqrt{1-y^2} \right], \\ x \geq 0, y \geq 0, x \leq y$$

$$\text{(iii)} \quad \tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}, \quad x \geq 0, y \geq 0$$

$$\text{I - 3 (i)} \quad \sin^{-1} \left(2x\sqrt{1-x^2} \right) = \begin{cases} 2 \sin^{-1} x & \text{if } |x| \leq \frac{1}{\sqrt{2}} \\ \pi - 2 \sin^{-1} x & \text{if } x > \frac{1}{\sqrt{2}} \\ -(\pi + 2 \sin^{-1} x) & \text{if } x < -\frac{1}{\sqrt{2}} \end{cases}$$

$$\text{(ii)} \quad \cos^{-1} (2x^2 - 1) = \begin{cases} 2 \cos^{-1} x & \text{if } 0 \leq x \leq 1 \\ 2\pi - 2 \cos^{-1} x & \text{if } -1 \leq x < 0 \end{cases}$$

$$\text{(iii)} \quad \tan^{-1} \frac{2x}{1-x^2} = \begin{cases} 2 \tan^{-1} x & \text{if } |x| < 1 \\ \pi + 2 \tan^{-1} x & \text{if } x < -1 \\ -(\pi - 2 \tan^{-1} x) & \text{if } x > 1 \end{cases}$$

$$\text{(iv)} \quad \sin^{-1} \frac{2x}{1+x^2} = \begin{cases} 2 \tan^{-1} x & \text{if } |x| \leq 1 \\ \pi - 2 \tan^{-1} x & \text{if } x > 1 \\ -(\pi + 2 \tan^{-1} x) & \text{if } x < -1 \end{cases}$$

$$\text{(v)} \quad \cos^{-1} \frac{1-x^2}{1+x^2} = \begin{cases} 2 \tan^{-1} x & \text{if } x \geq 0 \\ -2 \tan^{-1} x & \text{if } x < 0 \end{cases}$$

$$\text{If } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1} \left[\frac{x+y+z-xyz}{1-xy-yz-zx} \right] \text{ if, } x > 0, y > 0, z > 0 \\ \text{ \& } (xy + yz + zx) < 1$$

NOTE:

$$\text{(i)} \quad \text{If } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi \text{ then } x + y + z = xyz$$

$$\text{(ii)} \quad \text{If } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2} \text{ then } xy + yz + zx = 1$$

$$\text{(iii)} \quad \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$$